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Growth, Debt, and Sovereign Risk in a Small, Open Economy

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This paper develops a macroeconomic model for a small, open, developing economy that borrows abroad — to study the dynamic interaction between debt and growth and the impacts of various policies and exogenous shocks on the rate of capital accumulation, the current account, and debt. Adjustment policies that increase productivity and efficient use of capital increase both growth and the stock of external debt — but the new level of debt may be sustainable in the long run.

Most macrodynamic models of economies treat the supply of debt as infinitely elastic. Using a more realistic model, Bhandari, Haque, and Turnovsky have analyzed the dynamic consequences of various kinds of disturbances in a representative small developing economy that can borrow only at a premium and therefore faces an upward-sloping supply of debt. They conclude that:

- An upward shift in the supply of debt — which may be taken to represent an increase in the world interest rate — leads to a long-run decline in external debt, a higher domestic interest rate, less capital stock, and a reduced trade surplus. At some stage, the trade surplus becomes a deficit and more debt is accumulated.

- An increase in the marginal cost of debt (risk premium) — which almost certainly raises the long-run domestic interest rate and lowers the capital stock — may or may not lower long-run external debt as well. It depends on the ratio of the country's external debt to the net credit of its private sector.

- An increase in productivity raises the long-run stock of capital but leaves the level of external debt and the interest rate unchanged in the long run.

- Fiscal expansion (an increase in government spending) has almost no effect in either the short run or the long run. Consumers offset government spending by reducing private consumption. (This conclusion is based on the assumption that employment is fixed.)

Bhandari, Haque, and Turnovsky developed their macroeconomic model for a small, open, developing economy that borrows abroad — to study the dynamic interaction between debt and growth and the impacts of various policies and exogenous shocks on the rate of capital accumulation, the current account, and debt. They used the intertemporal optimizing representative agent model, which incorporates a realistic debt supply schedule and the risk premiums associated with lending to sovereign borrowers, which most models lack.

This paper is a product of the Macroeconomic Adjustment and Growth Division, Country Economics Department. Copies are available free from the World Bank, 1818 H Street NW, Washington DC 20433. Please contact Raquel Luz, room N11-057, extension 61588 (46 pages with tables).

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I. Introduction

The continuing debt crisis that many developing countries have faced in the current decade has underscored the need to understand the relationships between debt accumulation and growth, as well as the need to develop policy approaches that foster adjustment in the external account while maintaining, or preferably increasing, the growth of output. There is little doubt that developing economies require external capital since typically they cannot generate adequate resources domestically to achieve the growth rates that may lead to an improvement in living standards. Experience with external borrowing, however, has shown that repayments are not always easily made on time. This has led in some cases, to substantial adjustment efforts causing a significant cutback in domestic development efforts and hence a reduction in the rate of growth. Over-borrowing, resulting from inadequate perceptions of domestic growth potential, has occurred on occasion. Long gestation lags in investment projects, or the inability to adequately price domestic infrastructured projects that have been financed by external debt, or a combination of both, has led to difficulties in meeting repayment commitments in some cases (Kharas (1983), Kharas and Shishido (1986)). International capital markets are likely to react to their perception of a country's ability to repay and hence seek to reduce their exposure at a time when a country requires more capital (Sachs (1984)). The question of an appropriate debt strategy for sustained growth has therefore been an important one in recent years.

A related issue, that has also received considerable attention during this period of the debt crisis, is that of achieving a needed adjustment in the external accounts while sustaining, or even increasing, the growth momentum in the economy. When external resources are not available to the extent necessary, adjustment is clearly required in domestic spending and resource mobilization. Concessional external financing from multilateral agencies such as the Fund and the Bank is available to allow policy packages to be implemented to facilitate such adjustments (Khan et al). The desired policy mix for making such adjustments should be growth enhancing, or at least growth preserving.

The purpose of this paper is to develop a macroeconomic model for a small open developing economy that borrows abroad, in order to study the dynamic interaction between debt and growth, as well as the impacts of various policies and exogenous shocks on the rate of capital accumulation, the current account, and debt. The framework we employ is that of the intertemporal optimizing representative agent model, which has increasingly been used to advantage to analyze a variety of issues in international macroeconomics. 1/

Despite the current preoccupation with the debt crisis, there are few existing macroeconomic models which allow issues pertaining to it to be

1/ For recent examples of this approach applied to various types of exogenous and policy shocks see Brock (1987), Buiter (1984), Obstfeld (1982, 1988), Sen and Turnovsky (1989a, 1989b).

addressed adequately. 1/ This is because most of the existing literature is based on uncovered interest parity with the supply of debt to the economy being perfectly elastic. On the other hand, research has shown that the risks associated with sovereign borrowing does result in lenders charging a risk premium as well as, on occasion, cutting off credit to borrowers. 2/ Thus for a substantive analysis of debt related issues the model should include an appropriate debt supply schedule that incorporates the risk premia associated with lending to sovereign borrowers. The model presented below incorporates such a realistic debt supply schedule. 3/ Since fiscal policy is an important element in growth, debt accumulation and adjustment strategies, the model analyzes the impact of changes in government expenditure on the key macroeconomic variables. Specifically the model enables us to analyze the impact of both demand management and structural policies on variables such as the household consumption-saving decisions, the investment decisions of the firm, and the current account and hence debt accumulation.

1/ Reference should be made to a recent paper by Otani and Villanueva (1988) which analyzes the accumulation of capital and external debt in a neoclassical growth model. However, that paper adopts a very different approach and emphasizes different issues from those addressed in the present paper.

2/ See for example Eaton and Gersovitz (1981), Sachs (1984), Cooper and Sachs (1985). See Edwards (1984) for an empirical investigation of the risk premium.

3/ There are existing macroeconomic models which utilize a debt supply function similar to that used in this paper. Eaton and Turnovsky (1983) incorporate such a function in their analysis of exchange rate dynamics under covered interest parity. Obstfeld (1982) also considers an upward sloping debt function in his intertemporal optimizing analysis of terms of trade shocks.

The rest of the paper is organized as follows: Section II specifies the analytical framework. Section III discusses the long-run effects and dynamics of a balanced budget policy, while the debt-financed deficit policy is discussed in Section IV. Section V briefly discusses alternative specifications of the debt function. The concluding section summarizes the main results of the paper and discusses some policy implications of the analysis. For convenience, all technical matters relating to the solutions are discussed in the Appendix.

II. The Analytical Framework

1. The structure of the economy

The economy we shall consider comprises three sectors: (i) consumers; (ii) firms; and (iii) the government. For analytical tractability all consumers and firms are identical, enabling us to focus on the representative unit in each group. The economy produces a single traded good. The model is real, with the only financial asset held by domestic residents being a traded bond. International borrowing by the government is allowed, while consumers may borrow or lend to the government, which is largely consistent with practice in borrowing countries.

Unlike most macrodynamic models of the type considered here, our model incorporates issues of sovereign risk. Eaton and Gersovitz (1981) have shown that because of the moral hazard associated with sovereign risk, lenders will charge a risk premium on lending to countries, and may,

on occasion, also apply lending limits. ^{1/} This risk premium may vary directly with the stock of debt that the country holds since the probability of default is likely to increase with increased costs of servicing the debt, unless the costs associated with default also increase with debt accumulation. Sachs (1984), Sachs and Cohen (1982), and Cooper and Sachs (1985) have shown that a country, by adopting apparent growth-oriented policies, as well as policies that enhance its foreign exchange earning capacity, can shift the upward sloping debt supply curve outward, so that at each level of debt a lower risk premium is being charged. Thus, for example, larger investment efforts or the adoption of efficiency-enhancing structural adjustment packages will result in a lower evaluation of country risk by lenders. The main conclusion of this literature is that the level of indebtedness of the country is likely to determine both the extent of additional credit that is likely to be made available, as well as the terms on which it will be made available. This proposition is best formalized by an upward sloping supply curve.

To incorporate this feature of international borrowing, we assume that the interest rate, $i(z)$, charged on foreign debt, z , is:

$$i(z) = i_0 + i_1\omega(z) \qquad \omega'(z) > 0, \omega''(z) > 0 \qquad (1)$$

^{1/} Stiglitz and Weiss (1981) have shown that even in cases of individual borrowing because of informational asymmetries or problems associated with moral hazard, risk premia or credit constraints, or both, are known to exist.

where i_0 is the interest rate prevailing internationally and $i_1\omega(z)$ is the country specific risk premium which varies directly with the stock of debt held by the country. 1/ The possibility of a cutoff in debt, is captured by the assumption that the function $\omega(z)$ is convex ($\omega''(z) > 0$). In this case, the fact that the interest rate rises at an increasing rate with the level of indebtedness means that at some point it becomes prohibitive, so that an effective lending limit is reached. 2/ With this specification, an exogenous increase in foreign interest rates is captured by an increase in i_0 , while an exogenous shift in country-specific risk can be represented by a shift in the parameter i_1 . 3/

a. Consumers

Interest parity requires that the domestic interest rate be equalized with that facing the economy. Hence expression (1) above will also represent the interest rate prevailing at home, at which domestic consumers can borrow or lend to the government. With this background the representative consumer chooses his consumption and bond holdings to maximize the intertemporal utility function:

1/ Since the framework is designed to deal with debtor countries we shall assume throughout that $z > 0$.

2/ This enables us to depict the Eaton and Gersovitz (1981a,b) argument.

3/ This formulation which postulates the cost of debt to depend upon the absolute level of national debt is similar to that adopted by Eaton and Turnovsky (1983) and Obstfeld (1982). Alternative specifications which incorporate the endogeneity of country-creditworthiness as argued by Sachs (1984) and others will be considered later.

$$\int_0^{\infty} U(x, \bar{l}) e^{-\beta t} dt \quad U_x > 0, U_{xx} < 0 \quad (2)$$

subject to the budget constraint:

$$\dot{x} + \dot{b} = w\bar{l} + \pi + i(z)b - T \quad (3)$$

and the initial condition:

$$b(0) = b_0$$

where x is consumption, \bar{l} is labor supply assumed to be fixed, β is the consumer rate of time discount, w is the wage rate, π is profit distributed by firms to households, b is the stock of debt (or bonds) held by domestic residents, and T denotes lump sum taxes. 1/ The utility function is assumed to be concave. 2/ The present value Hamiltonian for the consumer problem may be written as:

$$H = e^{-\beta t} \{ U(x, \bar{l}) + \lambda [w\bar{l} + \pi + i(a-b)b - T - x - \dot{b}] \} \quad (4)$$

where a denotes the total stock of government debt ($a = z + b$), and λ is the Lagrange multiplier associated with the budget constraint. The

1/ For simplicity, labor is assumed to be fixed. Since in a developing country context the endogeneity of labor is not likely to be a critical issue, this assumption is not viewed as being particularly restrictive.

2/ If $b > 0$ the consumers are creditors while if $b < 0$ then they are debtors. An examination of the budget constraint shows that if consumers are creditors, then acquisition of increasingly costly debt by the government adds to disposable income and conversely.

required optimality condition for the individual's consumption decision is: 1/

$$U_x = \lambda \quad (5)$$

The optimal dynamic path is determined by the budget constraint, along with

$$\beta - \frac{\dot{\lambda}}{\lambda} = i - i'b \quad (6)$$

as well as the transversality condition:

$$\lim_{b \rightarrow \infty} \lambda b e^{-\beta t} = 0 \quad (7)$$

Equation (6) determines optimal bond accumulation by equating the marginal rate or return to consumers on consumption (the left hand side) to the marginal cost of an additional unit of debt facing consumers. 2/

b. Firms

Firms produce output using capital and the fixed supply of labor by means of the production function $y = f(k, \bar{l})$ which is assumed to possess standard neo-classical properties. Net profits of the firm at each point in time are, therefore, given by:

1/ Subscripts and primes (') denote derivatives.

2/ Note that the cost of debt depends on whether the consumer is a net creditor or a net debtor. In the former case the marginal cost exceeds the interest rate; in the latter case, the opposite is true.

$$\pi(t) = f(k, \bar{l}) - w\bar{l} - C(I) \quad (8)$$

where I is the rate of investment.

The function $C(I)$ represents the installation costs associated with the purchase of I units of new capital. It is assumed to be an increasing convex function of I : $C' > 0$, $C'' > 0$. In addition, we assume $C(0) = 0$, $C'(0) = 1$ so that the total cost of zero investment is zero, and the marginal cost of the initial installation is unity. This formulation of the installation function follows the original specification of adjustment costs introduced by Lucas (1967) and Gould (1968). ^{1/} More recent work by Hayashi (1982) and others postulates an installation function which depends upon k as well as I . This modification makes little difference to our analysis and for simplicity we retain the simpler specification.

The firm's optimization problem is to

$$\text{Maximize } \int_0^{\infty} \pi(t) e^{-\int_0^t i(s) ds} dt = \int_0^{\infty} [f(k, l) - wl - C(I)] e^{-\int_0^t i(s) ds} dt \quad (9)$$

subject to

$$\dot{k} = I \quad (10)$$

^{1/} Note that this specification implies that in the case where disinvestment may occur $I < 0$, $C(I) < 0$ for low rates of disinvestment. This may be interpreted as reflecting the revenue obtained as capital is sold off. The possibility that all changes in capital are costly can be incorporated by introducing sufficiently large fixed costs, so that $C(0) > 0$. This does not alter our analyses in any substantive way.

and the initial condition

$$k(0) = k_0$$

Two further points about this specification of the firm's optimization problem should be noted. First, (10) abstracts from physical depreciation. Second, the firm is assumed to finance investment purely from retained earnings and therefore does not borrow.

Writing the Hamiltonian function for firms as

$$H = e^{-\int_0^t i(s)ds} [f(k, \bar{l}) - w\bar{l} - C(I)] + qe^{-\int_0^t i(s)ds} (I - \dot{k}) \quad (11)$$

where q is the Lagrange multiplier associated with the accumulation equation (10), the optimality conditions are:

$$C'-(I) = q \quad (12)$$

$$f_k = -\dot{q} + i(z)q \quad (13)$$

$$w = f_l(k, \bar{l}) \quad (14)$$

together with the transversality condition

$$\lim_{t \rightarrow \infty} qke^{-\int_0^t i(s)ds} = 0 \quad (15)$$

Equation (12) determines the level of investment in each period by

equating the marginal cost of investment to the shadow price capital.

Equation (15) can be rewritten as:

$$\frac{f_k(k, \bar{l})}{q} + \frac{\dot{q}}{q} = i(z) \quad (16)$$

Here the left hand side, which equals the sum of the marginal physical product of capital deflated by q , and the percentage rate of change in the shadow price of capital, i.e., \dot{q}/q , is the rate of return on investing in a unit of capital. Optimality requires that this return should be equated, at each point in time to the interest rate. In view of the assumption about labor supply, equation (14) requires that the wage rate be set equal to the marginal physical product of the fixed supply of labor.

c. Government

The remaining agent in the domestic economy is the government, which operates in accordance with its budget constraint

$$\dot{a} = g + i(z)a - T \quad (17)$$

Thus, government spending, g , plus the interest obligations on outstanding government debt $i(z)a$, must be financed either by additional taxes, which we take to be lump sum, or by issuing additional debt. Of course, the government retains the choice of borrowing at home or abroad.

2. Macroeconomic equilibrium

Summing the consumers budget constraint (3), the firm's profit relation (8), and the government budget constraint (17), and noting that $\dot{z} = \dot{a} - \dot{b}$, the rate of decumulation of foreign debt, $-\dot{z}$, is shown to equal the current account balance. 1/

$$-\dot{z} = f(k) - x - C(I) - g - i(z)z \quad (18)$$

The current account balance is simply the sum of the trade balance, $f(k) - (x + C(I) + g)$, plus the service account which in this case is only the debt service, $-i(z)z$.

By combining the optimality conditions derived for the individual sectors above, together with the accumulation equations, the macroeconomic equilibrium may be summarized as follows:

$$U_x(x) = \lambda \quad (19a)$$

$$C'(I) = q \quad (19b)$$

$$\dot{\lambda} = \lambda[\beta - i(z) + i'(z)(a - z)] \quad (20a)$$

$$\dot{q} = i(z)q - f_k(k) \quad (20b)$$

$$\dot{k} = I(q) \quad (20c)$$

$$\dot{z} = x + C(I) + g - f(k) + i(z)z \quad (20d)$$

1/ From here on for convenience, labor will be suppressed.

$$\dot{a} = g + i(z)a - T \quad (20e)$$

The pair of static equations (19a), (19b) determine consumption as a function of the marginal utility of consumption, λ ,

$$x = x(\lambda) \quad x' < 0 \quad (21a)$$

and investment as a function of its shadow price q

$$I = I(q) \quad I' > 0 \quad (21b)$$

The latter equation will be recognized as being a Tobin q theory of investment. Substituting for x and I into (20a) - (20e) describes the dynamics of the economy.

It is well known that when capital is perfectly mobile internationally, so that the interest rate facing the small economy is fixed at the given world rate, that the dynamic structure simplifies drastically. In order for a steady-state equilibrium to exist in such an economy, the discount rate, β , must equal the world interest rate, and this implies that the marginal utility of consumption must remain constant, at $\bar{\lambda}$, say. The dynamics of the accumulation of capital k and its shadow price q , which constitute the core dynamics of the economy, are jointly determined by (20c) and (20b), respectively. The long-run equilibrium in such an economy has the property that it depends upon the initial stocks of real and financial assets. This in turn means that temporary shocks have permanent effects; see, e.g., Sen and Turnovsky (1989a) for an example of such a model.

The fact that the economy faces an upward sloping supply function for debt changes the dynamics in a fundamental way. With the domestic interest rate depending upon the stock of national debt, the discount rate β is no longer tied to the world interest rate, and the marginal utility of consumption, λ , is not constant. Instead, it depends upon the accumulation of both foreign debt z and government debt a , and thereby, the dynamics of the entire economy become highly interdependent.

In order to complete the specification of the dynamics, government budgetary policy needs to be specified. As a useful benchmark, which is analytically tractable, we shall focus first on the balanced-budget policy

$$\dot{a} = 0, T = g + i(z)\bar{a}$$

where lump sum taxes T are continually adjusted to finance expenditures. In Section IV below, we shall also discuss a form of debt financing. In order to be sustainable in the long run, this needs to be accompanied by a once-and-for-all change in lump sum taxes.

Using this model we shall analyze the long-run and short-run effects of a number of disturbances, focusing on the tradeoffs these incur on the rate of capital accumulation, on the one hand, and the accumulation of foreign debt, on the other.

III. Balanced Budget Policy

1. The long run

As noted in Section III.2 and shown in the Appendix, the dynamics of the system involves forward looking behavior. The short-run transition is

therefore determined in part by the long-run steady state. It is therefore convenient to begin with a consideration of the latter. Since the government is assumed to maintain a balanced budget, its stock of debt remains fixed at \bar{a} say.

The steady state of the economy is reached when all the variables cease to change, i.e., $\dot{\lambda} = \dot{q} = \dot{k} = \dot{z} = 0$. With no depreciation of capital, steady-state investment is zero, which in turn implies that the shadow price of investment, $q = 1$. Denoting the steady-state values by tildes, the long run can be described by:

$$U'(\bar{x}) = \bar{\lambda} \quad (22a)$$

$$i(\bar{z}) - i'(\bar{z})(\bar{a} - \bar{z}) = \beta \quad (22b)$$

$$f_k(\bar{k}) - i(\bar{z}) = i_0 + i_1\omega(\bar{z}) \quad (22c)$$

$$f(\bar{k}) - \bar{x} - g = i(\bar{z})\bar{z} \quad (22d)$$

$$\bar{T} = g + i(\bar{z})\bar{a} \quad (22e)$$

These equations jointly determine the steady-state values of the marginal utility $\bar{\lambda}$, consumption \bar{x} , capital stock \bar{k} , national debt \bar{z} , and lump sum taxes \bar{T} .

The equilibrium defined by (22a) - (22e) has a particularly simple recursive structure. The equilibrium level of the foreign debt \bar{z} is determined by (22b) which requires that consumers equate the costs or returns to them of buying or selling the marginal bond to their rate of time preference. Since the debt schedule facing the economy and hence the consumer, is upward sloping, the cost (or return) to holding a marginal

bond equals the interest rate plus the impact of the additional bond on the interest rate. Having determined \bar{z} , the domestic rate of interest is determined by the debt supply schedule. Equation (22c) then determines the long-run capital stock by equating the marginal physical product of capital to the domestic interest rate. At the same time, (22e) determines the required lump sum taxes necessary to finance government expenditures and interest on outstanding government debt. Equation (22d) is the steady-state balance of payments equilibrium condition. In the long run, the country must run a balance of trade surplus of sufficient magnitude to finance the interest on the outstanding foreign debt. Given the capital stock, and hence output, and the level of interest payments, this determines the level of consumption \bar{x} . Finally, given \bar{x} , the equilibrium marginal utility of consumption $\bar{\lambda}$ is determined by (22a).

Table 1 summarizes the long-run effects of shifts in the exogenous variables namely: the foreign interest rate, i_0 ; the risk premium, i_1 ; a productivity shock θ ; and an increase in government expenditure, g ; on the macroeconomic variables, i.e., the stock of debt, the capital stock, the domestic interest rate, the trade balance, consumption, and taxation. 1/

An increase in the level of international interest rates, represented by an upward shift in i_0 raises the marginal return or cost to consumers from holding an additional bond. Since in the long-run, this marginal return must equal the fixed rate of time discount β , this requires a

1/ The condition $2i' - i''\bar{b} > 0$ given in Table 1 is essentially a stability condition; see Appendix.

reduction in the former, to offset the effect of the higher i_0 , and maintain equality with the latter. This is brought about by a reduction in the level of external debt, \bar{z} . What this does to the domestic interest rate is not entirely clear. On the one hand, the upward shift in the debt supply schedule leads to a substitution away from foreign debt to domestic debt, putting upward pressure on the domestic interest rate. On the other hand, the reduction in the long-run stock of external debt lowers the risk premium. The overall effect depends upon which of these factors dominates and this in turn depends critically upon the nature of the risk premium function $\omega(z)$. Intuition would suggest that the domestic interest rate will, on balance, rise, and this in fact is likely to be so under realistic conditions. For example, if the debt supply schedule is linear ($i'' = 0$) or if domestic private residents hold no bonds ($\bar{b} = 0$), then a rise in the more costly foreign debt does indeed result in a rise in the domestic interest rate, though not by the full amount. In fact, a 1 percentage point increase in the foreign interest rate leads to a .5 percentage point increase in the domestic interest rate. If $i'' > 0$ and domestic private residents are net debtors ($\bar{b} < 0$), the upward pressure on the domestic interest rate is increased. On the other hand, if $i'' > 0$ and domestic private residents are larger creditors ($\bar{b} > 0$), the response of the domestic interest rate is reduced and indeed it may now fall. 1/ We view this however, as a highly unlikely scenario.

1/ Assuming a convex debt function of the form $i = i_0 + i_1 z^\alpha$, $\alpha > 1$, this will be so if the ratio $\bar{b}/\bar{a} > 1/\alpha$.

The response of the domestic capital stock depends upon that of the domestic interest rate. Taking the more plausible case where the latter rises, the equilibrium capital stock falls, while the financing requirement of the government rises. As noted, in the long run, the trade surplus must finance the interest costs $i(z)z$ of the foreign debt. While one effect of the higher foreign interest rate is to reduce the stock of foreign debt, this is likely to be offset by a higher domestic interest rate. The net effect on total interest payments is unclear and depends upon the function $\omega(z)$. In the case where this is linear, the decline in foreign debt dominates the higher domestic interest rate and overall interest payments decline. Assuming this case prevails, the long-run trade balance which was, and remains, in surplus to meet the economy's interest obligations, is reduced. What happens to domestic consumption is uncertain, even in the simplest case. For example, while the linear debt schedule leads to a reduction in capital stock and hence output, less output is now devoted to interest payments on international debt. ^{1/} The net effect on consumption depends upon which of these influences dominates. Moreover, since utility is a function of consumption, an

^{1/} These results may be usefully compared with the long-run effects of an increase in the foreign interest rate under the limiting assumption of uncovered interest parity. In such a model, the domestic interest rate rises by the same amount as does the foreign interest rate, leading to a larger fall in the domestic capital stock than in the present case. The stock of external debt can be shown to decline by an amount which is proportional to the reduction in the capital stock, with the resulting effect on the long-run trade balance being ambiguous, depending upon the stock of external debt.

upward shift in the cost of debt may, or may not, lead to a corresponding reduction in steady-state welfare.

Turning to the impact of an increase in the risk premium i_1 , we see that this will raise or lower the marginal return or cost to consumers from holding additional debt, according to whether $\omega - \omega' \bar{b} \gtrless 0$. In the case where domestic consumers are net debtors ($\bar{b} < 0$), this quantity is certainly positive, raising the marginal cost to them of incurring additional debt. As a result, they reduce their level of debt (i.e., increase \bar{b}) so as to maintain the equality between the marginal cost of debt and the fixed rate of time preference β . In this case, external debt will fall. However, if consumers are net creditors, the marginal return to holding more debt may either rise or fall with i_1 , even for the simplest debt function, depending upon the level of \bar{b} . In the case where \bar{b} is sufficiently small so that $\omega - \omega' \bar{b} > 0$, external debt will still fall. However, if \bar{b} is sufficiently large to reverse this inequality, a higher risk premium will lead to a higher equilibrium stock of external debt. ^{1/}

The response of the domestic interest rate depends upon whether the upward shift in the marginal cost of debt brought about by i_1 more than offsets the decline resulting from the likely reduction in the stock of external debt. Again this depends upon the form of the function $\omega(z)$. For plausible debt supply functions, such as the linear, or constant

^{1/} For example, for the constant elasticity convex debt function $i = i_0 + i_1 z^\alpha$, $\alpha \geq 1$, the quantity $\omega - \omega' \bar{b} = z^{\alpha-1} [z - \alpha \bar{b}]$. The criterion determining whether external debt falls or rises with i_1 depends upon whether the ratio of the country's external debt to the net credit of its private sector is greater or less than α , i.e., whether $z/\bar{b} \gtrless \alpha$.

elasticity ($i = i_0 + i_1 z^\alpha$, $\alpha \geq 1$) the interest rate will certainly rise. However, a decline cannot be ruled out, although it is even less likely to occur than in response to an upward shift in i_0 . The stock of capital moves counter to the interest rate and therefore, almost certainly will fall. The equilibrium trade balance will rise or fall to cover external payments commitments. Focusing on the linear debt supply function, if domestic residents are net debtors, external debt as well as interest payments decrease and therefore, the trade surplus declines; otherwise it could go in either direction. Finally, the effects on long-run consumption and utility are unclear, although both will decline if net interest payments increase.

Column 3 of Table 1 summarizes the effect of a productivity shock, θ , which is introduced as a shift operator in the production function. ^{1/} Such a disturbance has no effect on the steady state stock of external debt and the equilibrium trade balance too, remains unaffected. The effect on capital stock depends on the sign of $f_{k\theta}$, the impact of the productivity shock, θ , on the marginal physical product of capital. If the shock is productivity-enhancing i.e., $f_{k\theta} > 0$, the capital stock will rise; if not, it will fall. The effect on consumption depends on whether or not the shock increases output i.e., $f_\theta \geq 0$, and whether or not capital stock increases.

The last column of Table 1 summarizes the impact of a change in government expenditures. Since, in this model all government

^{1/} For this case, the production function is changed to $f(k, \theta)$.

expenditures are merely for consumption purposes, and the government budget is balanced, a change in government expenditure affects only the consumption of the private sector. Any increase (decrease) in government expenditures has to be matched fully by an increase (decrease) in taxation and hence a commensurate decrease (increase) in private consumption.

2. Transitional dynamics

The transitional dynamic behavior of the economy is determined by the system of differential equations, (14a) - (14g). The solution of this fourth order dynamic system is described in the Appendix. In the short run the stock variables, k and z may be regarded as predetermined. Any response to shocks will, therefore, impact on the two shadow prices, $q(0)$ and $\lambda(0)$. As shown in the Appendix, these prices are determined by the expected long run responses of both the capital stock and the foreign debt. In this respect, the dynamics of the system are forward looking.

In this section we will examine the transitional effects of exogenous shocks, namely, shifts in the debt schedule, productivity shocks, and changes in the government expenditure levels. The shocks that we will consider are all unanticipated permanent shocks.

a. Increase in foreign interest rate

The previous section showed that an increase in i_0 leads to a long-run decline in external debt and almost certainly in the stock of capital as well. Assuming this to be so, both of these long run effects contribute to an immediate fall in the shadow price of investment $q(0)$,

and therefore in the level of investment. ^{1/} The rate of capital accumulation, therefore slows down as a result of this shock. With the stock of foreign debt fixed instantaneously, a 1 percentage point increase in the foreign interest rate leads to a corresponding immediate 1 percentage point increase in the domestic interest rate although over time, the domestic rate will decline to the smaller steady-state response. The higher short term interest with the fixed stock of debt, means that the cost of servicing the debt also immediately increases, although it too declines over time, as both the interest rate and the stock of debt decline to their steady-state values.

The effect of an increase in i_0 , on initial consumption is unclear. Equation (5) shows that the consumption varies inversely with the marginal utility of consumption, λ . However, we are unable to establish unambiguously the initial response of $\lambda(0)$, and hence that of consumption. Various factors are in effect. First, the short-run marginal utility depends upon the long-run marginal utility, $\bar{\lambda}$, which may either increase or decrease, as previously discussed. Secondly, since we know that the long-run capital stock declines, capital will decumulate over the transition path. This would tend to increase consumption over the transitional path. An offsetting impact on consumption, however, obtains from the fact that long-run external debt is known to decline, and hence

^{1/} For obvious reasons we restrict our discussion to the plausible case where the increase in the foreign interest rate leads to an increase in the long-run domestic interest rate and a corresponding decline in the long-run capital stock. The perverse case, where the long-run interest rate declines can be analyzed similarly but is of little interest

the debt stock must also be reduced in the transition. Since the long-run debt of the government is known to be fixed at \bar{a} , the private supply of debt has to increase and this may occur at the expense of consumption.

The impact effect of the increase in external interest rates on the trade balance is in the direction of increasing the surplus. Although the debt servicing cost on impact increases and the effect on consumption is uncertain, it appears that the decline in investment alone is sufficient to increase (reduce) the surplus (deficit). Since output is fixed in the short run because of the cost of adjusting the capital stock, investment has to be reduced to a level that will allow enough of a change in the trade balance, in the direction of a surplus, to allow the immediate increase in the debt servicing commitments to be met as well as the debt stock to be reduced as required by long-run considerations.

The contrast between the short-run and long-run effects of an upward shift in the cost of debt can be usefully summarized at this point. An increase in i_0 leads to long-run declines in both the capital stock and the level of external debt. Despite a somewhat higher interest rate, the long-run cost of debt servicing declines and this requires a smaller trade surplus to finance. In the short-run, prior to any adjustment in the stock of debt, the domestic interest rate responds by the full amount of the increase in the foreign rate. Consequently the debt servicing requirements in the short-run are increased. At the same time, since higher interest rates in the long run lead to a reduced capital stock in the long run, investment declines. The cumulative effect of these changes

is that in the short run the trade balance improves to meet the increased interest payments, as well as to allow the stock of foreign debt to decline.

Further characterization of the transitional dynamic path followed by k and z can be obtained by using the dominant eigenvalue method due to Calvo (1987). Writing the solutions (A5), (A6) in the Appendix, for k and z in the form

$$k = \bar{k} + A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t}$$

$$z = \bar{z} + \phi(\mu_1) A_1 e^{\mu_1 t} + \phi(\mu_2) A_2 e^{\mu_2 t}$$

we have

$$\frac{z - \bar{z}}{k - \bar{k}} = \frac{\phi(\mu_1) A_1 e^{(\mu_1 - \mu_2)t} + \phi(\mu_2) A_2}{A_1 e^{(\mu_1 - \mu_2)t} + \phi(\mu_2)}$$

where $\phi(\mu_1) > 0$, $\phi(\mu_2) < 0$ are defined in the Appendix. Since μ_2 is the dominant stable root (i.e., $0 > \mu_2 > \mu_1$), it follows that as $t \rightarrow \infty$ $(z - \bar{z})/(k - \bar{k}) \rightarrow \phi(\mu_2) < 0$. That is, z and k asymptotically approach their respective steady-state values along a ray having a negative slope $= \phi(\mu_2)$. The initial phase of the path can be determined by evaluating dz/dk , and d^2z/dk^2 at the initial instant $t = 0$. In the present case of the upward shift in the cost of debt, i_0 , we have already

essentially shown that $dz(0)/dk(0) > 0$, and imposing additional weak conditions we can establish further that $d^2z(0)/Dk(0)^2 < 0$.

The transitional path followed by k and z is illustrated in Figure 1. The starting point is at the origin, with the new steady-state equilibrium being at A, having lower stocks of both physical capital and foreign debt. In the limit, this is approached along the locus XX having the negative slope $\phi(\mu_2)$. Initially the declining capital stock and external debt causes the system to move in a southwesterly direction from 0. The convexity of this locus, and the subsequent convergence along the negative ray implies that the stock of foreign debt overadjusts during the transition. After declining to the point B, z then begins to increase to its new equilibrium level. Intuitively, the convexity of the transitional locus OA implies an increasing decline in k relative to z over time. This in turn leads to an eventual decline in output, which exceeds the decline in other components of the current account. The initial current account surplus eventually becomes a deficit and at that time additional external debt is incurred. ^{1/}

b. Increase in risk premium

The qualitative short-run responses to an increase in the marginal cost of debt, i_1 , are given in Column 2 of Table 2. As before these responses also depend on the long-run response of the level of external debt, which as shown in the last section depends upon $(\omega'\tilde{b}-\omega)$. If

^{1/} This contrasts with the dynamics under the limiting case of uncovered interest parity when the paths followed by z and k can both be shown to be monotonic.

$\omega' \tilde{b} < \omega$, so that \tilde{z} falls, the results are qualitatively the same as those for an increase in i_0 and for essentially the same reasons. Once again the short-run interest rate exceeds the long-run. If, on the other hand, $\omega' \tilde{b} > \omega$, so that the long-run debt rises, short-run responses are less clear. Even though one can establish that there will be an initial decline in the shadow cost of capital $q(0)$, causing an immediate decumulation of capital, it is now possible for this to be accompanied by a current account deficit leading to an increase in foreign debt. In any event even if the external debt does not decline on impact, it will have to rise at some point during the transitional path.

c. Productivity shock

The productivity shock is simpler to analyze. The reason is that since it has no long run impact on national debt, the only long run response driving short run behavior is the change in the capital shock. In order to be concrete, assume $f_{k\theta} > 0$, i.e., the productivity shock impacts positively on the marginal physical product of capital. In this case, the increase in the long run level of the capital shock will lead to an instantaneous increase in the shadow price $q(0)$ causing investment demand to increase and an increased accumulation of capital to take place. Moreover, since consumption increases in the long run, the steady state value of the marginal utility of consumption, $\tilde{\lambda}$, is now lower. Instantaneously $\lambda(0)$ may rise, or more likely fall, though if so, by a smaller amount than in the long-run. Consequently, though consumption increases immediately it will not increase by the full amount of the

increase expected in the steady state. This is because as capital is accumulated over the transition, the expansion in the economy stimulates consumption.

There is no immediate effect on the interest rate or interest payments. The higher level of investment and the likely increase in consumption, leads to a deterioration in the trade balance and therefore in the current account. Foreign debt therefore begins to accumulate resulting in a rising interest rate. However, this is only temporary. Over time, as capital is accumulated, output increases. Eventually, output exceeds the levels of domestic demand by an amount in excess of that of additional interest payments leading to a surplus on the current account. At that time foreign debt is reduced and eventually returns to its original level. The adjustment path followed by capital and external debt is illustrated in Figure 2. The dynamics of debt are mirrored in the behavior of interest rates. Increasing current account deficits and accumulating debt lead to rising domestic interest rates on account of the increasing risk premium. When the current account switches from a deficit to a surplus and the process of debt reduction begins, interest rates start to decline, also returning to their pre-shock levels.

d. Increase in government expenditure

Since the long run stocks of \tilde{k} and \tilde{z} remain unaffected by changes in government expenditures and since the long run behavior of these state variables determine the dynamics, changes in government expenditures do not impact on any variable other than consumption and revenues. As

government expenditures increase (decrease) a corresponding increase (decrease) in revenues occurs, leading to a corresponding decrease (increase) in consumption.

IV. Debt-Financed Deficit Policy

Up to this point we have assumed that the government's budget is continuously balanced. This assumption is a simplifying one, made in part to maintain analytical tractability. 1/ The present section briefly considers a modification wherein the domestic government finances its deficit with the issuance of debt instruments. 2/

At the outset it may be noted that such a debt finance policy requires an accommodating one-for-all adjustment in the initial level of lump sum taxes, in order to be sustainable in the long-run. Formally, this arises from the fact with the stock of government debt constrained to adjust continuously, the number of unstable roots to the dynamic system (3) exceeds the number of "jump" variables (2). In order to obtain a viable solution therefore, an additional "jump" variable is required. This may be accomplished by appropriately choosing the initial level of lump-sum taxes. The steady-state of the model is now defined by:

$$U_x(\tilde{x}) - \tilde{\lambda} = 0 \quad (23a)$$

$$i_0 + i_1 \tilde{z} - i_1 (\tilde{a} - \tilde{z}) = i_0 + i_1 (\tilde{a} - 2\tilde{b}) = \beta \quad (23b)$$

1/ As discussed in the Appendix, the dynamic structure of the balanced-budget variant involves four differential equations. With debt finance, the resulting dynamics is of the fifth order; the additional source of dynamics is the evolution of the stock of government debt \dot{a} .

2/ The technical details are discussed in the Appendix.

$$f_k(\bar{k}) = i_0 + i_1 \bar{z} \quad (23c)$$

$$f(\bar{k}) - \bar{x} - g = (i_0 + i_1 \bar{z}) \bar{z} \quad (23d)$$

$$T = g + (i_0 + i_1 \bar{z}) \bar{z} \quad (23e)$$

$$\psi_1(\bar{z} - z_0) + \psi_2(\bar{k} - k_0) + \psi_3(\bar{a} - a_0) = 0 \quad (23f)$$

where equations (23a) - (23e) are identical to equations describing the steady-state for the balanced-budget case while (23f) is an additional relationship linking changes in government debt, total national debt and capital stock that as shown in the Appendix must additionally be satisfied. The coefficients ψ_i 's are defined as:

$$\psi_1 = \bar{a} i' [\mu_1 \mu_2 - I' f'' - i(\mu_1 + \mu_2 - i)] > 0$$

$$\psi_2 = \bar{a} i'^2 I' \phi(\mu_1) \phi(\mu_2) < 0$$

$$\psi_3 = -(\mu_1 - i)(\mu_2 - i)(\mu_1 + \mu_2 - i) > 0$$

where in turn μ_1, μ_2 are the stable (negative) roots of the dynamic system (see Appendix for details) while $\phi(\mu_i)$ are defined as:

$$\phi(\mu_i) = \frac{\mu_i(\mu_i - i) + f'' I'}{I' i'}$$

The steady-state stock levels of \bar{k} , \bar{z} and \bar{a} are jointly determined by equation (23b), (23c) and (23d) and depend (via (23f)) upon the initial conditions k_0 , a_0 and z_0 . Because of this, even temporary changes can give rise to permanent effects. Once \bar{k} and \bar{z} are determined as above,

the balance of payments condition (23d) determines steady-state consumption \bar{x} while $\bar{\lambda}$ may now be obtained from (23a). The required accommodation in lump-sum taxes is determined from (23e). Once T is set in this manner, it is not thereafter adjusted and the subsequent deficit is bond financed with the stock of government bonds following a second-order adjustment path. The short-run and long-run effects of the various disturbances can be analyzed in a manner similar to that previously and may be briefly summarized. For expositional convenience we restrict our remarks to the case where the debt schedule is linear.

An upward shift in the cost of debt, i_0 , will lower both the steady-state stock of foreign debt and physical capital, just as it did before. The long-run interest rate will also increase though by less than the increase in i_0 . Long-run government debt may either rise or fall. This is because the higher interest rate, accompanied by the lower foreign debt may, or may not, raise the marginal return (or cost) of purchasing an additional bond to consumers. If it does, \bar{a} will have to rise, in order to maintain this marginal return equal to the fixed rate of consumer discount β . But if this marginal return falls, then \bar{a} will have to rise for the same reason.

The short-run responses to an upward shift in the debt schedule can be analyzed as before. Again we can establish that the long-run declines in \bar{k} and \bar{z} generate short-run decumulations in these quantities. The interest rate rises more in the short run and the higher short-run

interest costs to the government, causes a short-run increase in the rate of accumulation of government debt.

While the changes in the marginal cost of debt i_1 and government expenditure give rise to virtually the same responses as before, more substantive differences arise with respect to a productivity shock. Not only does it lead to a long-run increase in capital stock as before, but now this is accompanied by higher long-run levels of both national and government debt. The higher marginal productivity of capital resulting from the increase in productivity raises the domestic interest rate, thereby encouraging more borrowing from abroad. The higher level of external debt raises the marginal rate of return of additional bonds to domestic consumers, and government debt will have to rise in order to maintain equality with β . The initial responses are also in the direction of the long-run effects.

V. Alternative Specifications of the Debt Function

As noted earlier, it has been argued that country creditworthiness may not be a function of only the stock of debt alone. Factors that serve to increase productivity or growth, or foreign exchange earnings, may all serve to indicate an increased capacity to service debt, and hence shift the debt supply schedule outwards. To incorporate this idea the debt function can be altered in at least two interesting ways. Note that the above argument basically requires debt to be scaled in some way in its effect on the rate of interest. The central point appears to be that creditors are not as concerned with the absolute amount of debt held by

the country as with its ability to service the debt, which is better represented by debt relative to some measure of servicing capacity. According to this line of reasoning, alternative specifications might be to use either debt-output (z/y) or debt-capital ratio (z/k) in (1) above. 1/

With employment fixed and output dependent on capital alone, these two modifications are almost identical. Moreover, neither of them alters our results in any fundamental way.

For example, if one were to use the relationship

$$i(z/k) = i_0 + i_1 \omega(z/k)$$

and assumes a balanced government budget, one finds that the two critical steady-state equilibrium relationship (22b), (22c) are modified to

$$i(\tilde{z}/\tilde{k}) - i'(\tilde{z}/\tilde{k})(\bar{a}-\tilde{z})/\tilde{k} = \beta \quad (22b')$$

$$f_k(\tilde{k}) = i_0 + i_1 \omega(\tilde{z}/\tilde{k}) \quad (22c')$$

The interdependence between z and k in the determination of the interest rate now breaks down the earlier recursivity in the long-run equilibrium. These two equations now jointly determine the steady-state stocks of capital and external debt. The changes in these equilibrium levels then

1/ This would transform (1) to

$$i(z/y) = i_0 + i_1 \omega(z/y) \quad (1')$$

$$i(z/k) = i_0 + i_1 \omega(z/k) \quad (1'')$$

respectively.

drive the short-run dynamics as before. Generally, the qualitative behavior of the system is little changed. The only point worth noting is that a productivity disturbance will now affect the steady state stocks of both debt and capital.

VI. Conclusion

Although it has long been recognized that small developing economies can borrow only at a premium and therefore face an upward sloping supply of debt, most existing macrodynamic models of such economies treat the supply of debt as being infinitely elastic. This paper has departed from this assumption and analyzed the dynamic consequences of various kinds of disturbances in an economy facing an upward sloping supply of debt. The presence of such a constraint changes the dynamics in fundamental ways from those which occur under the more usual, but less realistic, assumption of perfect capital mobility.

In order to highlight the issues involved, we have based most of our analysis on the simplest such model and then modified it in alternative ways. The basic model is one in which the government maintains a balanced budget, the cost of debt rises with its absolute level, and employment remains fixed. We have emphasized the tradeoffs which exist between the rate of capital accumulation, on the one hand, and the rate of accumulation of foreign debt, on the other, and analyzed how these respond to the various exogenous disturbances.

In this basic model, the only sources of disturbances which incur dynamic tradeoffs are disturbances either to the supply of debt function,

or the production function. To some extent, the effects on the economy are sensitive to the form of the debt supply function, in particular its degree of convexity, as well as the net asset position of domestic private residents. The main findings, summarized as follows, are based on what we view as being the most plausible assumptions with respect to the aspect:

- (1) An upward shift in the supply of debt, which may be taken to represent an exogenous increase in the world interest rate, leads to a long-run decline in external debt, a partially higher domestic interest rate, and a lower capital stock. The combination of the lower external debt and the only partial response of the domestic interest rate results in a decline in the long-run cost of servicing the external debt, implying a reduction in the long-run trade surplus. In the short run, the domestic interest rate responds fully to the higher foreign interest rate, causing total interest costs to rise. The rate of investment falls, generating a sufficient increase in the trade surplus for the stock of external debt to begin to decline, along with the capital stock. The transitional path for foreign debt is non-monotonic. At some stage, the trade surplus becomes a deficit and further debt is accumulated as the new steady-state equilibrium is approached.
- (2) An increase in the marginal cost of debt (risk premium), while almost certainly raising the long-run domestic interest rate and lowering the capital stock, may or may not, lower long-run

external debt as well. Whether the stock of debt is reduced or not depends upon the ratio of the country's external debt to the net credit of its private sector. If this ratio is greater than some critical value (which depends the form of the debt supply function), then foreign debt is reduced in the long run and vice versa. The trade balance adjusts in the direction of the surplus of the long-run impact of the shock on debt accumulation is positive; i.e., if the country is not heavily indebted. Otherwise, the effect is uncertain.

- (3) A positive productivity disturbance, resulting from say structural efficiency-enhancing measures, will raise the long-run stock of capital, while leaving the long-run level of external debt and interest rate unchanged. In the short run, since capital is now more productive, it begins to accumulate. A current account deficit is therefore generated and foreign debt is accumulated, leading to a higher interest rate. The accumulation of debt, however, is only temporary. Eventually as output increases, the current account deficit turns to a surplus and debt, along with the interest rate, returns to its original level.
- (4) A fiscal expansion, taking the form of an increase in government expenditure has virtually no effect, either in the short run or in the long run. All that occurs is that consumers immediately

offset the additional government expenditure with an equal reduction in private consumption.

Most of these conclusions carry over to the more general models, though with modifications in some instances. For example, in the case of a bond-financed deficit, the only substantive change that occurs is that a productivity shock now has a long-run effect on the level of external debt as well. Increased capital accumulation as a result of a positive productivity disturbance will now be accompanied by both higher stocks of external and government debt in the long run.

Modifying the debt supply function and specifying it in terms of debt relative to some measure of the country's economic performance such as output or capital stock also leaves things mostly unchanged. Again the main difference is that the long-run stock of foreign debt will no longer be independent of the productivity shock. As capital or output increases, the marginal cost of debt declines, allowing a higher long-run stock of debt.

One important feature of our results is the limited role for government expenditure policy. This is a direct consequence of the assumption that employment is fixed. As a result, both the long-run stock of debt and the long-run stock of capital are determined independently of government expenditure. Since it is the changes in these long-run equilibrium stocks which drive the short-run dynamics, the latter are also independent of government expenditure. Endogenizing employment increases the role for government expenditure policy. While the long-run capital-

labor ratio remains independent of government expenditure, the long-run capital stock itself, and therefore the short-run dynamics do respond to this form of fiscal policy.

In conclusion, perhaps it is worthwhile to summarize some implications of this analysis for growth and adjustment. The model discussed in this paper, suggests that demand-management approaches to adjustment which seek to reduce the fiscal deficit, while reducing debt accumulation, are likely to have a negative impact on growth. Policies for structural adjustment that enhance productivity and the more efficient utilization of capital will increase capital accumulation and hence growth, but these policies will also increase the stock of external debt. The increase in debt may only be transitory if the debt supply function depends upon the absolute level of debt, rather than on its level relative to the capacity of the economy. But even if the debt supply function is such as to lead to higher long-run stock of debt, this increased level of debt will nevertheless be sustainable in the long run.

1. Dynamic properties with balanced budget

Under the assumption of a balanced budget the dynamic structure of the system (20) is a fourth order system, which may be expressed in linearized form about the steady-state equilibrium

$$\begin{pmatrix} \dot{\lambda} \\ \dot{q} \\ \dot{k} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -\lambda(2i' - i''b) \\ 0 & i & -f_{kk} & i' \\ 0 & I' & 0 & 0 \\ x' & I' & -f_k & i + i'z \end{pmatrix} \begin{pmatrix} \lambda - \bar{\lambda} \\ q - \bar{q} \\ k - \bar{k} \\ z - \bar{z} \end{pmatrix} \quad (A1)$$

where the elements appearing in the matrix are evaluated at steady state. The dynamic properties of the economy depend critically upon the eigenvalues of the characteristic equation of (A1), namely

$$\frac{\mu(\mu-1)}{\mu(\mu-1) - [i'z\mu + x'a_{14}]} = \frac{\mu(\mu-1) + f_{kk}I'}{I'i'} = \phi(\mu) \quad (A2)$$

where the element $a_{14} = -\lambda(2i' - i''b)$.

Assuming $2i' - i''b > 0$, the following properties can be established:

- (1) The product of the four roots is positive, implying that there are either 0, 2 or 4 positive roots.
- (2) The sum of the roots is positive, ruling out the case of 0 positive roots.
- (3) The coefficient of μ in (A2) is negative, ruling out the possibility of all roots being positive.

We are therefore left with the case of 2 positive and 2 negative roots, which may be ordered as follows

$$\mu_1 < \mu_2 < 0 < \mu_3 < \mu_4 \quad (A3)$$

The dynamics is therefore a saddlepoint, with the stock variables k and z evolving gradually at all time, and the shadow prices λ , q being allowed to undergo instantaneous jumps in response to new information.

The quantities $\phi(\mu_1)$, $\phi(\mu_2)$, as defined in (A2) are both critical parts of the solution. By direct evaluation of the characteristic equation, one can establish

$$\phi(\mu_1) > 0 > \phi(\mu_2) \quad (A4)$$

where μ_1 , μ_2 , are ordered as above.

We shall focus our analysis on stable adjustment paths beginning from given initial capital stock k_0 and stock of national debt z_0 . The solutions for k , z , λ , and q along such paths are

$$k = \tilde{k} + \frac{[\phi(\mu_2)d\tilde{k}-d\tilde{z}]}{\phi(\mu_1)-\phi(\mu_2)} e^{\mu_1 t} + \frac{[d\tilde{z}-\phi(\mu_1)d\tilde{k}]}{\phi(\mu_1)-\phi(\mu_2)} e^{\mu_2 t} \quad (A5)$$

$$z = \tilde{z} + \frac{\phi(\mu_1)[\phi(\mu_2)d\tilde{k}-d\tilde{z}]}{\phi(\mu_1)-\phi(\mu_2)} e^{\mu_1 t} + \frac{\phi(\mu_2)[d\tilde{z}-\phi(\mu_1)d\tilde{k}]}{\phi(\mu_1)-\phi(\mu_2)} e^{\mu_2 t} \quad (A6)$$

$$\lambda = \tilde{\lambda} + \frac{a_{14}\phi(\mu_1)}{\mu_1} \left[\frac{\phi(\mu_2)d\tilde{k}-d\tilde{z}}{\phi(\mu_1)-\phi(\mu_2)} \right] e^{\mu_1 t} + \frac{a_{14}\phi(\mu_2)}{\mu_2} \left[\frac{d\tilde{z}-\phi(\mu_1)d\tilde{k}}{\phi(\mu_1)-\phi(\mu_2)} \right] e^{\mu_2 t} \quad (A7)$$

$$q - \tilde{q} + \frac{\mu_1}{I'} \left[\frac{\phi(\mu_2) d\tilde{k} - d\tilde{z}}{\phi(\mu_1) - \phi(\mu_2)} \right] e^{\mu_1 t} + \frac{\mu_2}{I'} \left[\frac{d\tilde{z} - \phi(\mu_1) d\tilde{k}}{\phi(\mu_1) - \phi(\mu_2)} \right] e^{\mu_2 t} \quad (A8)$$

where $d\tilde{k} = \tilde{k} - k_0$, $d\tilde{z} = \tilde{z} - z_0$ denote the long-run changes in k and z from their respective initial starting points.

One can eliminate $e^{\mu_1 t}$, $e^{\mu_2 t}$, to define the two locuses

$$\lambda - \tilde{\lambda} = \frac{-a_{14}}{\phi(\mu_1) - \phi(\mu_2)} \left[\frac{\phi(\mu_2)}{\lambda_2} - \frac{\phi(\mu_1)}{\mu_1} \right] (z - \tilde{z}) - \frac{a_{14} \phi(\mu_1) \phi(\mu_2)}{\phi(\mu_1) - \phi(\mu_2)} \left[\frac{1}{\mu_1} - \frac{1}{\mu_2} \right] (k - \tilde{k}) \quad (A9)$$

$$q - \tilde{q} = \frac{\mu_1 - \mu_2}{I' [\phi(\mu_1) - \phi(\mu_2)]} (z - \tilde{z}) + \frac{\mu_2 \phi_1(\mu_1) - \mu_1 \phi_2(\mu_2)}{I' [\phi(\mu_1) - \phi(\mu_2)]} (k - \tilde{k}) \quad (A10)$$

These are both three dimensional planes relating the respective instantaneous shadow prices to the slowly evolving dynamic variables k and z . They are the analogues to the usual two dimensional stable adjustment paths associated with saddlepoints.

The solutions reported in equations (A5) - (A8) form the basis for the analysis of the short-run dynamics in response to the various disturbances presented in Section IV. The different shocks identified differ simply in terms of how they impact on the long-run equilibrium stock of capital and national debt.

2. Dynamic properties with debt-financed deficit

When the government's budget is permitted to be imbalanced via the

use of debt financing, the dynamics of the economy are governed by a fifth-order system which may be stated in linearized form as follows:

$$\begin{pmatrix} \dot{\lambda} \\ \dot{q} \\ \dot{k} \\ \dot{z} \\ \dot{a} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -\lambda(2i' + i''b) & \lambda i' \\ 0 & i & f_{kk} & i' & 0 \\ x' & I' & 0 & 0 & 0 \\ x' & I' & f_k & (i + i'z) & 0 \\ 0 & 0 & 0 & \tilde{a}i' & i \end{pmatrix} \begin{pmatrix} \lambda - \tilde{\lambda} \\ q - \tilde{q} \\ k - \tilde{k} \\ z - \tilde{z} \\ a - \tilde{a} \end{pmatrix} \quad (A11)$$

This system may be shown to have two stable roots ($\mu_1, \mu_2 < 0$) and three unstable roots ($\mu_3, \mu_4, \mu_5 > 0$). Analogous to (A5) - (A8), the stable solution, starting from initial stocks k_0, z_0 is

$$k = \tilde{k} + A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t} \quad (A13)$$

$$z = \tilde{z} + \phi(\mu_1) A_1 e^{\mu_1 t} + \phi(\mu_2) A_2 e^{\mu_2 t} \quad (A14)$$

$$\lambda = \tilde{\lambda} + \frac{i'}{\mu_1} \left[\frac{a_{14}}{i'} + \frac{\lambda i' \tilde{a}}{\mu_1 - i} \right] \phi(\mu_1) A_1 e^{\mu_1 t} + \frac{i'}{\mu_2} \left[\frac{a_{14}}{i'} + \frac{\lambda i' \tilde{a}}{\mu_2 - i} \right] \phi(\mu_2) A_2 e^{\mu_2 t} \quad (A15)$$

$$q = \tilde{q} + \frac{\mu_1}{I'} A_1 e^{\mu_1 t} + \frac{\mu_2}{I'} A_2 e^{\mu_2 t} \quad (A16)$$

$$a = \tilde{a} + \frac{\tilde{a}i'}{\mu_1 - i} \phi(\mu_1) A_1 e^{\mu_1 t} + \frac{\tilde{a}i'}{\mu_2 - i} \phi(\mu_2) A_2 e^{\mu_2 t} \quad (A17)$$

where $\phi(\mu_1)$ and a_{14} are defined as previously and for notational convenience A_1 and A_2 are given by

$$A_1 = \frac{\phi(\mu_2)d\tilde{k}-d\tilde{z}}{\phi(\mu_1)-\phi(\mu_2)} ; \quad A_2 = \frac{d\tilde{z}-\phi(\mu_1)d\tilde{k}}{\phi(\mu_1)-\phi(\mu_2)} \quad (A18)$$

The assumption that the stock of government debt evolves continuously from the initial level a_0 , can be shown to imply the relationships

$$\psi_1(z(t)-z_0)+\psi_2(k(t)-k_0)+\psi_3(a(t)-a_0)=0 \quad (A19)$$

$$\psi_1(\tilde{z}-z_0)+\psi_2(\tilde{k}-k_0)+\psi_3(\tilde{a}-a_0)=0 \quad (A19')$$

where

$$\psi_1 = \tilde{a}i'[\mu_1\mu_2 - I'f_{kk} - i(\mu_1 + \mu_2 - i)] > 0 \quad (A20)$$

$$\psi_2 = \tilde{a}i'^2I'\phi(\mu_1)\phi(\mu_2) < 0 \quad (A21)$$

$$\psi_3 = -(\mu_1 - i)(\mu_2 - i)(\mu_1 + \mu_2 - i) > 0 \quad (A22)$$

The significance of (A19) is that the dynamics of government debt becomes tied to that of k and z . There are in fact only two linearly independent unstable roots. As a further consequence, (A19') imposes an additional constraint on the steady state equilibrium, one which involves the initial points k_0 , z_0 and a_0 .

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Table 1. Long-Run Effects

		Increase in			
		i_0	i_1	θ	g
National debt	\tilde{z}	$-\frac{1}{D}$	$\frac{\omega' \tilde{b} - \omega}{D}$	0	0
Capital stock	\tilde{k}	$-\frac{(i' - i'' \tilde{b})}{\delta_{kk} D}$	$\frac{i'(\omega' \tilde{b} + \omega) - i'' \omega \tilde{b}}{f_{kk} D}$	$\frac{f_{k\theta}}{f_{kk}}$	0
Interest rate	\tilde{i}	$\frac{i' - i'' \tilde{b}}{D}$	$\frac{i'(\omega' \tilde{b} + \omega) - i'' \omega \tilde{b}}{D}$	0	0
Trade balance		$-\frac{[i - \tilde{z}(i' - i'' \tilde{b})]}{D}$	$\frac{id\tilde{z}}{di_1} + \frac{z d\tilde{i}}{di_1}$	0	0
Consumption	\tilde{x}	$\frac{f_k(i' - i'' \tilde{b})}{f_{kk} D}$ $+\frac{i - \tilde{z}(i' - i'' \tilde{b})}{D}$	$\left(\frac{i}{f_{kk}} - \tilde{z}\right) \frac{d\tilde{i}}{di_1}$ $-\frac{id\tilde{z}}{di_1}$	$f_\theta - \frac{f_k f_{k\theta}}{f_{kk}}$	-1
Lump sum taxes	\tilde{T}	$\frac{\bar{a}(i' - i'' \tilde{b})}{D}$	$\bar{a} \frac{d\tilde{i}}{di_1}$	0	1

$$D = 2i' - i'' \tilde{b} > 0$$

Table 2. Qualitative Short-Run Effects

		Increase in			
		i_0	i_1	θ	g
Debt accumulation	$\dot{z}(0)$	-	< 0 if $\frac{d\tilde{z}}{di_1} < 0$? if $\frac{d\tilde{z}}{di_1} > 0$	+	0
Investment	$\dot{k}(0)$	-	-	+	0
Interest rate	$i(0)$	$+(-1)$	$+\omega(\tilde{z})$	0	0
	$\frac{di(0)}{dt}$	-	?	+	0
Trade balance		+	< 0 if $\frac{d\tilde{z}}{di_1} < 0$? if $\frac{d\tilde{z}}{di_1} > 0$	-	0
Consumption	$x(0)$?	?	?	$-(-1)$
Lump sum taxes	$T(0)$	$+(-\bar{a})$	+	0	$-(-1)$

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